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On one simple and analytical "photon-migration" solution in 1-D light scattering theory and its consequences for the laser medical diagnostic problems

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ABSTRACT

It is well known that the exact and analytical theoretical solution of any physical tasks can be a powerful instrument for the analysis. But it is well known too, that in the general light transport and scattering theories, which are always used in different laser medical applications, there are only few exact and analytical approaches to solve the important modeling tasks. It can be shown that the conventional mathematic theory of the Markov processes can also provide some exact and analytical solutions for a number of practically important cases. As an example, in this report the analytical solution of 1-D pure scattering task with the use of the Markov processes formalism is presented. Some consequences of that for the general light scattering theory and for the noninvasive medical diagnostic problems are discussed as well. For instance, this solution can predict an enhanced value of the experimentally estimated transport scattering coefficient if the thin sample of biotissue is used. For the laser Doppler medical flowmetry in the case of a strong scattering media and a low level power of the laser our result can predict the appearing of additional spectra of the tissue's output signal which can be wrong interpreted like a conventional Doppler spectrum.

Keywords: photon, light, scattering, transport theory, Markov process, noninvasive diagnostics, medicine.

1. INTRODUCTION

The possibilities of the noninvasive laser and optic diagnostic technique in medicine are studied now very intensively[1-5]. Recently, in our previous research[6-8], it was shown that the efficiency of noninvasive optical diagnostic equipment in main manner depends on the efficiency of its mathematical software, which must exactly resolve the inverse optical task of the light scattering media theory. In turn, the exactness of the inverse optical algorithms in main manner depends on the exactness of the direct algorithms calculating a scattered light distribution into the turbid medium and on its surface. So, for the effective development of the data processing software for noninvasive optical diagnostic systems it is necessary to have some effective approaches to resolve a direct optical task. The best way to have one is the searching an exact and analytical solution of the direct task. Moreover, it is well known fact that the exact theoretical solution of a modeling physical task can be the most powerful instrument for the further analysis of particularities of the investigated process. However, in the classical optics of scattering media[9] there is the well-known opinion that only the "Milne's approach" leads to the exact analytical solution in a number of particular cases. Recently papers[10,11] presented some new ways to the analytical solution of the total scattering task. It can be shown that the conventional mathematic theory of the Markov processes can also provide some exact and simple analytical solutions for a number of practically important cases. And they, in turn, can provide some interesting consequences for the general light scattering theory and for noninvasive medical diagnostic problems as well.

2. GENERAL THEORY

An assumption of the separated photon migration phenomena in scattering media is now widely used in a lot of biomedical optic applications, especially in statistical Monte Carlo computer simulation. If this model is assumed then the probability of the photon location at any point "x_i" of the space can be interpreted like a probability of the state of the Markov N-section chain. In this case the magnitudes of the parameter "t" (time) in a function of conditional probability of transitions became quasi-discontinuous and the possibility to replace them by the test's numbers "*n*" (or "*m*", "*l*", etc.) appears. In such approach the common probability of the photon transition - $p(n,x_k|l,x_i)$ - from "x_i" point to "x_k" point obeys the general Markov equation[12]:

$$p(n,x_k|l,x_i)=\sum_j p(n,x_k|m,x_j)\,p(m,x_j|l,x_i) \qquad (\mathrm{l}<\mathrm{m}<\mathrm{n}). \tag{1}$$

For the homogeneous Markov chain the probabilities of transitions using "s" steps (s=n-l) form a matrix of probabilities:

$$p(x_k|s,x_i)\equiv \mathbf{p_{ik}(s)}, \tag{2}$$

where, in a conventional case, the sum of matrix's elements is:

$$\sum_{k=1}^{N} p_{ik}=1 \quad , \tag{3}$$

when N is a total number of chain's states which are taken into account.

The equation (2) shows that the probabilities of transitions with the use of any "s" steps can be ultimately represented using only one-step probabilities. In fact, it reduces a solution of the Markov process task to forming one-step matrixes and making the formal multiplications of them.

Let's take into account, as an example, a simple 1-D task of the photon wandering into a system of "N" reflecting walls (see fig.1). Let's consider the same coefficient of reflection – "R" – for each separate wall as well. Because of R≤1 it can be interpreted as a probability of the photon transition from one state to another when photon is reflected by wall. Whilst the probability of the opposite event for photon (crossing a wall) will be equal "1-R".

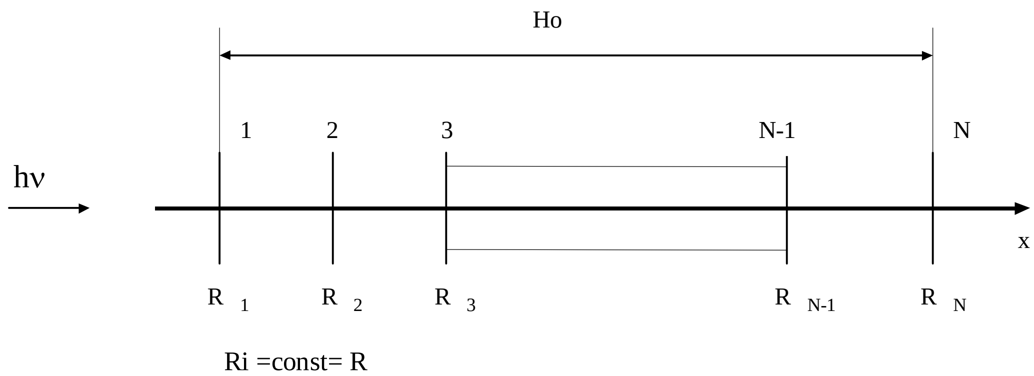

Fig.1 The scheme of the investigated pure scattering medium.

If we will find the total probability for the photon backscattering (or transmitting) by this layered medium then we will have a total amount of backscattered (transmitted) photons under real illumination, i.e. we will have a general solution of the investigated task.

For the correct use of the Markov process's mathematical formalism it is necessary to numerate all photon's states in a model system. If the simplest numeration will be chosen: before all walls – the state number i=1, between first and second walls – the state number i=2, between second and third walls i=3, etc., then one could determine all one-step probabilities for photon migration. Excepting states i=1 and i=N+1 (after all walls), the transition from any "i" state to a state i=i+1 or i=i-1 has a probability "1-R". The probability of staying in a state "i" is "R". If photon goes out of the medium (i=1 or i=N+1) the probability of changing its state falls down to the zero (there isn't any "back" process). But for this simplest numeration platform the Markov properties of a process are violated: the transition from "i" state to "i+1" or "i-1" state depends on a prehistory of the moving (was photon moved from left to right or from right to left). And the task can't be resolved using Markov's formalism.

In our investigation we have considered much more useful numerating platform which allowed us, none the less, to reduce this task to the Markov process problem. The even numbers (i=2,4,6,…) were used for the numeration of the photon states between walls and out of the walls when photon had motions from right to left. The uneven numbers (i=1,3,5…) were used when photon moved from left to right. Initial photons for that numeration platform had a state i=1, backscattered photons had a state i=2 and transmitted - i=2·N+1. Strictly speaking, on our numerating platform for the any even state “i” (excepting i=2) only states “i-1” or “i-2” are permitted for transition with the probabilities “R” and “1-R” respectively. For any uneven state “i” (excepting i=2·N+1) the permitted states are “i+1” and “i+2” with the same probabilities “R” and “1-R”. Any transitions from i=2 or i=2·N+1 states to any another states are not permitted (probabilities of transitions equal zero). After creating such statistical scheme of the task everyone can start a general determination.

In instance, for the problem of the laser noninvasive medical diagnostics with the use of backscattered light to register diagnostic data the total probability $P_S(N)$ of the photon transition from the state i=1 to state i=2 through any “s” numbers of steps (s=1,2,…∞) is interesting for us as a function of a number of walls “N”. It means that we must find the unlimited sum:

$$P_S(N) = \sum_{S=1}^{\infty} p_{12}^{N}(S) \quad , \tag{4}$$

where $P_S(N)$ is the total probability of the photon transition from the state i=1 to state i=2 through any “s” numbers of steps. The matrix of all one-step transitions “$\boldsymbol{\pi}_1$”, in instance for N=2, is a matrix $\mathbf{p}_{ik}^{N=2}(\mathbf{S}=\mathbf{1})$ with dimension 6x6:

$$\boldsymbol{\pi}_1 = \mathbf{p}_{ik}^{2}(\mathbf{1}) = \begin{pmatrix} 0 & R & 1-R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 1-R & 0 \\ 0 & 1-R & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-R & R & 0 \end{pmatrix} . \tag{5}$$

The probabilities of reaching the state “i=2” from the state “i=1” with the use of any “s” steps can be calculated by multiplication of corresponding matrixes (5). For example, for N=2 the corresponding probabilities are:

s=1: $p_{12}(1)=R$;
s=2: $p_{12}(2)=0$;
s=3: $p_{12}(3)=R(1-R)^2$;
s=4: $p_{12}(4)=0$;
s=5: $p_{12}(5)=R^3(1-R)^2$;
s=6: $p_{12}(6)=0$;
s=7: $p_{12}(7)=R^5(1-R)^2$
…………………………..

and the sum of them is the series:

$$P_S(2) = \sum_{S=1}^{\infty} p_{12}^{2}(S) = R + R(1-R)^2 + R^3(1-R)^2 + \ldots \tag{6}$$

Looking at (6) one can find that starting from the second item of the series there is, in (6), the trivial geometrical progression. So, the limit for (6) is trivial as well:

$$P_S(2) = \frac{2R}{1+R} \quad . \tag{7}$$

Taking into account layered medium systems with N=3,4,5…. it is easy to find that the general probability $P_s(N)$ as a general solution of our task for the backscattered light is:

$$P_S(N) = \frac{NR}{1+(N-1)R} \quad . \tag{8}$$

It must be mentioned that (8) is an exact and analytical solution for the pure scattering case and for a backscattered light. The backscattered flux (F_{BS}) can be calculated from (8) as:

$$F_{BS} = F_0 \cdot P_S(N), \tag{9}$$

where F_0 is the initial flux illuminating the turbid medium.

For a transmitted light or for another medium scheme the same way of calculation can be easily used and the total analytical solution can be easily yielded as well.

3. CONSEQUENCES

The first and very interesting consequence of (8) for the total theory of light propagation in turbid media relates to the question of the right understanding of the transport scattering coefficient term in a general transport theory. The Kubelka-Munk approach[9] allows everyone to obtain an exact and analytical solution of the 1-D pure scattering task (without light absorption into a medium) as well. That gives the following formula for the backscattering flux for our 1-D scattering medium:

$$F_{BS} = F_0 \cdot \frac{SH_0}{1+SH_0}, \tag{10}$$

where: “S” is the transport scattering coefficient for this turbid medium and “H_0” is the total thickness of the medium. When N>>1 from (8) and (10) one can yield the following relationship:

$$S = NR / H_0 \ . \tag{11}$$

So, the transport scattering coefficient is an effective and generalizing parameter of the task describing a radiance scattering by a total volume of the medium and depending on a total medium thickness in that number. Therefore, it can be proposed from (11), that the measurements of optical transport parameters *“S”* for different biological tissues when the different samples of tissues with the different thickness are used (the conventional laboratory technology) must give different results if inhomogeneties of the medium are distributed in it not quite uniform. This consequence is indirect confirmed by a lot of reports concerning the real optical properties of biological tissues estimated by the experimental study[13]. Moreover, we can predict now, that the less H_0 of the sample is used the more magnitude of “*S* “could be obtained from the experimental data. And vice versa. So for in vivo measurements in real clinical practice the optical scattering coefficient for a real skin can be less, than one for a thin sample of the skin.

The second interesting consequence is situated in the noninvasive laser Doppler flowmetry field. The presented equation (8) was yielded with the use of assumption of the possibility of innumerable photon migration steps “s” and innumerable amount of initial photons. That is correct for the infinite time of the signal registration and very big initial power of laser radiation only. If the power of light source is limited and a photodetector has a limited time response then the detected amount of photons will be different from sampling to sampling because of a stochastic nature of the migration process. It will cause of amplitude modulation of the measured photocurrent that, in instance, for the laser Doppler flowmetry equipment can be interpreted like an additional spectra of the tissue’s output signals or a “false” Doppler spectrum namely. So, only the strong scattering properties of tissues can be reason of the “Doppler” spectra in noninvasive flowmetry technology. Particularly it is good confirmed by remarks of the existence of a “biological zero phenomenon” in the laser Doppler diagnostic technique[14].

4. CONCLUSION

It is well known that the exact and analytical theoretical solution of any physical task can be a powerful instrument for the analysis. But it is well known too, that in general light transport and scattering theories, which are always used in the laser medical applications (especially noninvasive diagnostic applications), there are only a few exact and analytical approaches. That is why the analytical analysis of the majority of practically important cases of the laser light interaction with biotissues is today very difficult. It was shown in this report, that the Markov processes mathematical theory could be a useful instrument to obtain some exact solutions of the photon stochastic migration task. For example, presented simple solution for a pure scattering 1-D process allows researcher to estimate more correct physical means of the used transport coefficients in the transport theory, especially of the scattering transport coefficient, that is important for the way of looking for an exact approaches to resolve a direct optical task in the laser medical application field.

For instance, this solution can predict an enhanced value of the experimental estimated transport scattering coefficient if the thin sample of biotissue is used. For the laser Doppler medical flowmetry in the case of a strong scattering media and a low level power of the laser our result can predict the appearing of additional spectra into the tissue's output signal which can be wrong interpreted like a conventional Doppler spectrum.

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