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Scattering of Electromagnetic Waves by a Randomly Rough Surface as a Boundary Problem of Laser Radiation Interaction with Light-Scattering Materials and Media

D. A. Rogatkin

Moscow State Institute of Electronics and Mathematics, Moscow, 109028 Russia

e-mail: laserrog@mtu-net.ru

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Abstract—The light scattering by a rough surface with random Gaussian fluctuations of roughness is studied in the case of coarse roughness, whose parameters—mean deviation and correlation length—are much greater than the radiation wavelength. Closed analytical solutions of the problem are presented in terms of radiophysics for the boundary conditions of an ideal conductor and the impedance boundary conditions. These solutions are formulated in terms of a photometric scattering indicatrix. The possibility of their application to the problems of photometry and theory of radiative transfer and scattering in turbid media, in particular, in simulation of the process of boundary scattering of laser radiation by rough surfaces of biological tissues and media, is discussed.

INTRODUCTION

The recent development of laser noninvasive diagnostics in biology and medicine [1] has rekindled interest in the theoretical description of optical radiation propagation through turbid scattering materials and media. First of all, this is connected with the need to develop high-level algorithms for the processing of diagnostic data based on the solution of inverse problems of optics of light-scattering media [2]. The classical theory of radiative transfer in randomly inhomogeneous media [3], which is the most widely used today and is practically the only existing analytical method, does not always prove to be efficient in practice. For example, multidimensional spatial scattering problems and problems involving the calculation of the radiation field in close-packed media with pronounced scattering anisotropy present serious difficulties in classical radiative transfer theory (RTT) [4]. In connection with this, the development of new algorithms and methods for solving such problems is quite urgent.

The possibility of developing new analytical approaches to solving multidimensional problems of radiation propagation in the mass of a medium on the basis of Kubelka–Munk flow models has been demonstrated recently [5]. To set the boundary values of the flows, these models should use some function of the radiation intensity distribution at the interface between media, and a procedure for determining this function in the general case is not yet known. At present, the overwhelming majority of existing models in RTT employ local conditions of a plane interface. The boundary values of the radiation intensity in them are determined on the basis of the known Fresnel equations for an infinite plane medium–air interface. For the diffusion approximation, the zero flow of the diffusion component of radiation near the medium boundary is additionally specified [3]. Sometimes, more complex approximating equations are used for the reflection coefficient or some effective (conditional) interface between the two media is introduced [6].

Meanwhile, it is obvious that the surface of most real light-scattering materials and media, in particular, biological ones, is not planar, but rather loose (uneven) and randomly rough with the size of roughness, especially in the case of biological tissues, greater than the wavelength of the incident radiation. Intuitively, it is clear that the spatial parameters of the light beam upon its interaction with such a rough interface undergo certain changes as compared to the situation of radiation incidence on a smooth surface. Most likely, the beam penetrating into the medium will additionally broaden due to the surface roughness, and this initial broadening will affect, in some way, the general solution of the problem of radiation propagation inside the medium. However, classical RTT gives no answer to the question: How do the spatial parameters of the beam change as it crosses an uneven interface?

The neglect of rough boundaries in RTT seems rather surprising, especially in view of the fact that, in parallel with RTT, there are well-known diffraction methods of electrodynamics, which, in principle, allow such problems to be solved. These methods, developed initially for radar [7], have been repeatedly applied to problems of optics and photometry, for example, for explanation of the observed optical properties of construction materials [8]. Unfortunately, for many reasons, they still have not found a proper application to RTT problems. Thus, because these methods are radiophysical in origin, many solutions given by them are formulated in terms of electrodynamics rather than photometry. This complicates their direct application to RTT problems. As a consequence, for example, the scattering indicatrix in [9] has to be treated as the relative angular distribution of the radiation intensity with respect to the amplitude of the specular maximum, while, by photometric definition, the indicatrix must also bear absolute information about the radiation intensity in any considered direction [10]. In addition, because of certain mathematical difficulties, illustrative analytical solutions in the theory of diffraction of electromagnetic waves on randomly rough surfaces could be obtained until recently only for ideally conducting or well-conducting (metal) surfaces (the so-called Leontovich boundary conditions [11]), whose application in optics and photometry is quite limited at first sight. The relatively recent paper [12] has opened ways to the search for analytical solutions for dielectric surfaces and media, including most biological tissues, which, in the optical region, are dielectrics. The use of laser radiation in modern photometry and medicine has eliminated the last distinction between the computational methods of radar and optics in terms of coherence and other wave properties of the initial (incident) radiation.

Thus, there are now certain prerequisites for invoking the diffraction methods of electrodynamics to solve computational boundary problems of RTT. In particular, the following questions are considered: How does the rough surface of biological tissues scatter radiation as applied to computational RTT problems, and is it possible to describe (estimate) this scattering within the framework of the classical theory of diffraction of electromagnetic waves by randomly rough surfaces?

BASIC THEORY FOR IDEAL CONDUCTOR

It is convenient to begin the study of this problem by considering the simplest classical problem of diffraction of electromagnetic radiation by the surface of a rough ideal conductor. This allows us to obtain the basic reference solutions, which can then be further extended, and to analyze them for application to photometry.

In the general case, the overwhelming majority of problems in the modern theory of wave scattering on randomly rough surfaces are based on solution of the Green's integral vector equations, which are also known as the Stratton–Chu vector equations in diffraction problems of electrodynamics [13],

$$\begin{aligned}\mathbf{E}^S(\mathbf{r}) &= \operatorname{curl}\int_S \mathbf{n}'\times\mathbf{E}(\mathbf{r}')\varphi(\mathbf{r},\mathbf{r}')dS' \\ &+ \frac{i}{k\varepsilon}\operatorname{curl}\operatorname{curl}\int_S \mathbf{n}'\times\mathbf{H}(\mathbf{r}')\varphi(\mathbf{r},\mathbf{r}')dS', \\ \mathbf{H}^S(\mathbf{r}) &= \operatorname{curl}\int_S \mathbf{n}'\times\mathbf{H}(\mathbf{r}')\varphi(\mathbf{r},\mathbf{r}')dS' \\ &- \frac{i}{k\mu}\operatorname{curl}\operatorname{curl}\int_S \mathbf{n}'\times\mathbf{E}(\mathbf{r}')\varphi(\mathbf{r},\mathbf{r}')dS',\end{aligned} \tag{1}$$

where $\mathbf{E}^S(\mathbf{r})$ and $\mathbf{H}^S(\mathbf{r})$ are the complex amplitudes of the electric and magnetic fields scattered by the surface at the point $\mathbf{r}$, S is the area of the scattering surface, $\mathbf{n}'$ is the unit vector of the external normal to S at the surface point $\mathbf{r}'$, k is the wave number, ε and μ are the permittivity and permeability of the medium, and $\varphi(\mathbf{r},\mathbf{r}') = e^{ik|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$ is the Green's function.

Equations (1) are valid for any point of space with no restrictions. They allow calculation of the field scattered by the surface S through calculation of the components ($\mathbf{n}'\times\mathbf{E}$; $\mathbf{n}'\times\mathbf{H}$) of the field vectors tangential to S. Their exact values, in turn, can be found only through solution of additional integral equations, which, in the general case, is a rather difficult problem for arbitrary surfaces and media. Therefore, the tangential components are determined using various approximate methods. One of the simplest methods employs the boundary conditions of an ideal conductor

$$\mathbf{n}'\times\mathbf{H}(\mathbf{r}') = \mathbf{J}(\mathbf{r}'), \quad \mathbf{n}'\times\mathbf{E}(\mathbf{r}') = 0 \tag{2}$$

and the well-known Kirchhoff approximation:

$$\mathbf{J}(\mathbf{r}') = 2\mathbf{n}'\times\mathbf{H}^i(\mathbf{r}'), \tag{3}$$

where $\mathbf{H}^i(\mathbf{r}')$ is the field of the incident plane wave and $\mathbf{J}(\mathbf{r}')$ is the surface current on S. In this case, the system (1) becomes much simpler, and for the field scattered by the surface of an ideal conductor, it is sufficient to consider the integral equation

$$\mathbf{H}^S(\mathbf{r}) = \int_S \nabla_r\varphi(\mathbf{r},\mathbf{r}')\times[2\mathbf{n}'\times\mathbf{H}^i(\mathbf{r}')]dS'.$$

Its solution can be found in the closed form. It is only necessary to describe the rough surface itself and to choose the general geometry of the problem in the appropriate coordinate system. The general geometry of our problem is shown in the figure. To describe the randomly rough surface, from the viewpoint of obtaining the final analytical solution, it is most convenient to consider the roughness heights as a randomly Gaussian, homogeneous, and isotropic field $z = \xi(x, y)$ with zero mean ($\langle\xi(x, y)\rangle = 0$), variance $\langle\xi^2(x, y)\rangle = h^2$, and Gaussian correlation function

$$C(\tau) = \frac{\langle\xi(x, y)\xi(0, 0)\rangle}{\langle\xi^2(x, y)\rangle} = e^{-\tau^2/T^2},$$

where $\tau^2 = x^2 + y^2$ is the distance between the points considered and T is the correlation length.

Clearly, real roughness of both metal and dielectric surfaces can differ from this idealized model. However, as was shown in [14], the use of other correlation functions in the calculations only slightly affects the final result, at least for the field scattered into the outer half-space. However, this strongly complicates the procedure of derivation of the final analytical equations. Therefore, the concept of Gaussian roughness is, in our opinion, the most convenient for performing tentative calculations. Then we can change the parameters of the random field and estimate the differences in the results. It should be noted, however, that, in [15], on the basis of analysis of a large number of real rough surfaces, it was shown that a Gaussian random field fairly adequately describes the majority of such surfaces.

In our computational scheme, it is convenient to represent the surface as a square plate with side length L, illuminated by a unit unbounded linearly polarized plane electromagnetic wave incident at the angle φ to the normal

$$\mathbf{H}^i(\mathbf{r}) = \boldsymbol{\eta} e^{i(\mathbf{k}, \mathbf{r})}, \quad |\mathbf{k}| = k,$$

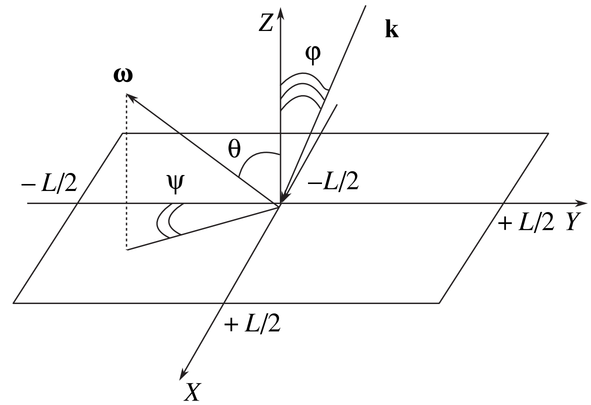
where $\boldsymbol{\eta}$ is the unit basis vector of the plane of polarization of the incident wave.

The direction of scattering into the outer half-space will be characterized by the vector $\boldsymbol{\omega}$. Assume that the central plane of the plate coincides with the coordinate plane XOY and consider the solution of the problem for the field scattered into the outer half-space [9]. The solution is constructed for the far zone of the radiation in the form

$$\mathbf{H}^S(\mathbf{r}) = \frac{ik}{2\pi}\frac{e^{ikR}}{R}\int_S \boldsymbol{\omega} \times [\mathbf{n}' \times \boldsymbol{\eta}] e^{i(\mathbf{q}, \mathbf{r}')} dS', \qquad (4)$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}\omega$ and R is the distance from the origin of coordinates to the observation point.

Then, using the Isakovich–Beckman formalism [9, 16], neglecting the boundary effects, averaging the scattered field over the ensemble of random surfaces, and summing the mean intensity of the coherent and incoherent parts of the scattered field, we can obtain the general solution of the problem in the form of an equation for the mean radiophysical intensity of the scattered field in the far zone normalized to the intensity of the initial incident wave [17],



General geometry of the problem (see explanations in the text).

$$H_1(\varphi, \theta, \psi) = \left|\frac{\mathbf{H}^S(\mathbf{r})}{\mathbf{H}^i(\mathbf{r}')}\right|^2 = \frac{1}{R^2}\frac{S}{\lambda^2}e^{-g}|\boldsymbol{\omega} \times [\mathbf{q} \times \boldsymbol{\eta}]|^2\frac{1}{q_z^2}$$
$$\times\left(S\left(\frac{\sin q_x L/2}{q_x L/2}\right)^2\left(\frac{\sin q_y L/2}{q_y L/2}\right)^2 + \pi T^2\sum_{m=1}^{\infty}\frac{g^m}{m!m}e^{-q_{xy}^2 T^2/4m}\right), \qquad (5)$$

where q_x, q_y, and q_z are the components of the vector $\mathbf{q}$; $q_{xy} = \sqrt{q_x^2 + q_y^2}$, $S = L^2$, $g = h^2 q_z^2$, and $\lambda = 2\pi/k$ is the wavelength.

For the optical wavelength region and for applications of this solution to problems of photometry and RTT, it should be reformulated in optical and photometric terms. The fundamental photometric term is the light strength or intensity $I_{e\lambda}$ [18], which is the angular distribution of the light flux $F_{e\lambda}$ from a point source [19] in some solid angle:[1]

$$I_{e\lambda} = dF_{e\lambda}/d\Omega, \qquad (6)$$

where $d\Omega$ is the element of the solid angle.

However, in photometric practice, relative, rather than absolute, measurements of the radiative fluxes are used in most cases, because this significantly simplifies the problem. For this purpose, the concept of the scattering (reflection) indicatrix is introduced. For identical illumination conditions, the indicatrix characterizes, in relative units, the angular distribution of the intensity of the scattered or reflected radiation with respect to some known easily modeled reference field [10]. Thus, to describe the volume scattering, as the normalizing ref-

[1] Hereinafter, for brevity and clarity, the subscripts of the photometric energy e and the wavelength λ are omitted.

erence field, photometry employs the field scattered from an ideal diffuse scatterer. This scatterer scatters the incident radiation uniformly over all directions with the mean spherical intensity $F_0/4\pi$, where F_0 is the incident flux [19]. By definition, the scattering indicatrix for an ideal scatterer is independent of the scattering angle and equal to a constant (unity). For description of the processes of surface reflection, photometry usually uses, as a reference, the so-called Lambertian surface illuminated normally ($\varphi = 0$). This surface forms the reflected field according to the Lambert law [10]

$$I(\varphi = 0, \theta, \psi) = I_0 \cos\theta,$$

where I_0 depends on the incident flux F_0, and, in the general case of loss-free reflection, it is equal to [20]

$$I_0 = F_0/\pi. \qquad (7)$$

This parameter, i.e., the amplitude of the radiation intensity formed by the Lambertian surface in the direction normal to the surface under the illumination conditions presented above, is used as the normalizing intensity when determining the reflection indicatrix. Thus, the reflection indicatrix of any unknown reflector, $\rho_r(\varphi, \theta, \psi)$, is characterized by the equation [17]

$$\rho_r(\varphi, \theta, \psi) = I_{\text{ref}}(\varphi, \theta, \psi)/I_0 = I_{\text{ref}}(\varphi, \theta, \psi)\pi/F_0, \qquad (8)$$

where $I_{\text{ref}}(\varphi, \theta, \psi)$ is the experimentally recorded reflected intensity.

The indicatrix of the Lambertian surface is equal to $\cos\theta$. At the same time, in electrodynamics, the radiation flux dF through an area element on the surface of a sphere, formed by an elementary source S located at the center of the sphere, is described by the well-known equation [11]

$$dF = \frac{1}{2} W_e |\mathbf{H}(\mathbf{r})|^2 dA, \qquad (9)$$

where W_e is the wave resistance of the medium and dA is the elementary area of the sphere's surface.

Since Eq. (5) is obtained for the far zone of the radiation, the scattering surface S in this problem can be considered as a point source. Then, comparing Eqs. (6) and (9) and using the equation for the solid angle[2]

$$d\Omega = \frac{dA}{R^2}\Omega_0, \qquad (10)$$

where R is the radius of the sphere corresponding to the distance to the receiver ($\Omega_0 = 1$ sr), we can easily obtain an equation relating the photometric and radiophysical radiation intensities,

$$I_{\text{ref}}(\varphi, \theta, \psi) = W_e |\mathbf{H}(\mathbf{r})|^2 R^2/2\Omega_0. \qquad (11)$$

[2] See Appendix 1.

This equation allows us to express the reflection indicatrix through the corresponding radiophysical field strengths. Since the flux F_0 incident on the surface S is determined by the equation

$$F_0 = \frac{1}{2} W_e |\mathbf{H}^i(\mathbf{r}')|^2 S,$$

using Eqs. (5), (8), (11) and omitting $\Omega_0 = 1$ sr, we can easily find[3] [16]

$$\rho_r(\varphi, \theta, \psi) = \frac{|\mathbf{H}^S(\mathbf{r})|^2 R^2 \pi}{|\mathbf{H}^i(\mathbf{r}')|^2 S} = \frac{\pi R^2}{S} H_1(\varphi, \theta, \psi). \qquad (12)$$

To check the validity of expression (12), we can try to model the Lambertian reflector by an ideally conducting rough plate. This modeling is quite justified, because in electrodynamics both the Lambertian reference (by definition) and the ideally conducting plate fully reflect (scatter) the incident radiation (owing to their electromagnetic properties). Moreover, this modeling is especially interesting because the Lambert law is still considered as one of the least theoretically justified laws in modern physics [22].

For modeling, consider the structure of solution (5). It consists of two terms, one of which is nonlinear with respect to S and λ and responsible for coherent scattering at low roughness. Another term, in contrast, is linear with respect to S and characterizes the incoherent random scattering at high roughness. It is clear a priori that, to obtain more or less isotropic scattering, as is required in the case of Lambert reflection, the roughness should be coarse enough. This leads to the condition $g \gg 1$ and the possibility to calculate the sum in Eq. (5) by the saddle point method [23]. Neglecting the first term, which is much smaller at $g \gg 1$, and using Eq. (12) and the explicit equations for the components of the vectors $\boldsymbol{\omega}$, $\mathbf{q}$, and $\boldsymbol{\eta}$ determined by the general geometry of the problem, we can obtain the following equation for the reflection indicatrix of a normally illuminated ideally conducting plate with coarse roughness:

$$\rho_r(\varphi = 0, \theta, \psi) = \frac{T^2}{4h^2(1+\cos\theta)^2} e^{-\frac{\sin^2\theta T^2}{4h^2(1+\cos\theta)^2}}. \qquad (13)$$

As an example, Table 1 illustrates the model indicatrices calculated by Eq. (13) for different T/h in comparison with the Lambertian indicatrix ($\cos\theta$). It can easily be seen that the Lambertian reflection is achieved almost ideally at $T/h = 4$. An increase in T/h changes the character of the reflected field, making it more specular.

[3] More rigorously, in the sense of radiophysics (by analogy with the definition of scattering cross section), Eq. (12) should be written [21] in the form $\rho_r(\varphi, \theta, \psi) = (\pi R^2/S) \lim_{R\to\infty} |\mathbf{H}^S(\mathbf{r})|/|\mathbf{H}^i(\mathbf{r}')|$, where the limit sign denotes the fulfillment of the condition of the far zone.

Table 1. Reflection indicatrix calculated by Eq. (13) for ideally conducting randomly rough surfaces at different observation angles θ and different *T*/*h* in comparison with the Lambertian indicatrix

θ, deg	Reflection indicatrix calculated by Eq. (13) for different parameters of surface roughness				Lambertian indicatrix (cos θ)
	T/*h* = 2	*T*/*h* = 4	*T*/*h* = 6	*T*/*h* = 8	
0	0.2500	1.0000	2.2500	4.0000	1
10	0.2519	0.9848	2.1325	3.5933	0.9848
20	0.2577	0.9388	1.8082	2.5859	0.9397
30	0.2673	0.8620	1.3545	1.4568	0.8660
40	0.2808	0.7550	0.8759	0.6160	0.7660
50	0.2981	0.6211	0.4712	0.1828	0.6428
60	0.3185	0.4686	0.1991	0.0343	0.5000
70	0.3401	0.3125	0.0606	0.0035	0.3420
80	0.3590	0.1737	0.0116	0.0001	0.1736
90	0.3679	0.0733	0.0011	0.0000	0

The slight difference of the indicatrix with *T*/*h* = 4 from cos θ for large viewing angles can be explained by the neglect of radiation shading and re-reflection on the surface in this formulation of the problem, whereas these factors are obviously present in the case of coarse roughness. Thus, the diffraction approach can be quite efficient in photometric problems; in particular, it allows an adequate description of such a model phenomenon as the Lambertian character of reflection.

MEDIA WITH FINITE CONDUCTANCE

Now we can pass on to the more general case of media with finite conductance. For these media, the initial integral equations (1) are to be solved. Thus, for the strength of the scattered magnetic field, we have to solve the equation

$$\mathbf{H}^S(\mathbf{r}) = \frac{ike^{ikR}}{4\pi R}\int_S e^{i(\mathbf{q},\,\mathbf{r}')}\{\boldsymbol{\omega}\times\mathbf{n}'\times\mathbf{H}(\mathbf{r}') + (\boldsymbol{\omega}\boldsymbol{\omega}^T - 1)\mathbf{n}'\times\mathbf{E}(\mathbf{r}')\}dS'. \quad (14)$$

This can be done most easily for the case of a medium with a high conductance, when the Leontovich impedance boundary conditions can be formulated on the rough surface of a plate [24],

$$\mathbf{n}\times\mathbf{E} = \sqrt{\frac{\mu}{\varepsilon}}\mathbf{n}\times\mathbf{n}\times\mathbf{H}. \quad (15)$$

These conditions reflect the fact that the tangential components of the field on the surface of a good conductor are continuously transformed into the transverse components of the field of the wave propagating deep into the conductor [11]. At $\sqrt{\mu/\varepsilon} \longrightarrow 0$, condition (15), as could be expected, transforms into the condition of an ideal conductor. In the case of the Kirchhoff approximation, for example, for the vertical polarization of the incident radiation, the tangential components of the fields on *S* can be expressed from condition (15) as follows:

$$\mathbf{n}\times\mathbf{H} = \frac{2}{1+\sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\cos\varphi}}\mathbf{n}\times\mathbf{H}^i,$$
$$\mathbf{n}\times\mathbf{E} = \frac{2}{1+\sqrt{\frac{\varepsilon}{\mu}}\cos\varphi}\mathbf{n}\times\mathbf{E}^i, \quad (16)$$

where φ is the local angle of incidence, and $\mathbf{H}^i$ and $\mathbf{E}^i$ are the vectors of the field of the incident wave.

Further simplification of the solution of Eq. (14) can be connected with the replacement of the coefficients depending on the local angle of incidence in Eq. (16) by their average values. Substituting these averages into Eq. (14), we can obtain a solution analogous to solution (5) for an ideal conductor, except for the modified factor $|\boldsymbol{\omega}\times\mathbf{q}\times\boldsymbol{\eta}|^2$. For example, for unpolarized light, Eq. (5) will include the sum

$$\left|\frac{\boldsymbol{\omega}\times\mathbf{q}\times\boldsymbol{\eta}_1}{1+\frac{1}{\cos\varphi}\sqrt{\frac{\mu}{\varepsilon}}} + \frac{(\boldsymbol{\omega}\boldsymbol{\omega}^T-1)\mathbf{q}\times\mathbf{e}_1}{\cos\varphi\sqrt{\frac{\varepsilon}{\mu}}+1}\right|^2 + \left|\frac{\boldsymbol{\omega}\times\mathbf{q}\times\boldsymbol{\eta}_2}{1+\frac{1}{\cos\varphi}\sqrt{\frac{\mu}{\varepsilon}}} + \frac{(\boldsymbol{\omega}\boldsymbol{\omega}^T-1)\mathbf{q}\times\mathbf{e}_2}{\cos\varphi\sqrt{\frac{\varepsilon}{\mu}}+1}\right|^2 \quad (17)$$

in place of the vector product. In Eq. (17), the subscripts 1 and 2 denote the vertical and the horizontal polarization, **e** is the unit vector of the electric field strength, and φ is the nonlocal angle of incidence. As for the rest, the form of solution (5) and, correspondingly, all of its statistical properties remain unchanged. It is interesting to note that, for the case of normal illumination of the surface, the polarization of the radiation is unimportant, and the sum (17) can be significantly simplified and expressed in explicit form. At $\mu = 1$ and with the use of the complex refractive index of the medium ($N = n + i\chi$), it takes the form

$$\frac{(1 + \cos\theta)^2[(n-1)^2 + \chi^2]}{(n+1)^2 + \chi^2}. \tag{18}$$

Thus, the equation for the reflection indicatrix in the case of a well-conducting and very rough surface is completely identical to Eq. (13), except for an extra factor—the Fresnel reflection coefficient. The angular characteristics of the scattered field remain unchanged, because in Eq. (5) at normal light incidence the vector product $|\boldsymbol{\omega} \times \mathbf{q} \times \boldsymbol{\eta}|^2$ for an ideal conductor has the form $(1 + \cos\theta)^2$.

Biological tissues and media are, certainly, not media with high conductance in the optical wavelength region. Typical values of the real and imaginary parts of the complex refractive index for them can be found in numerous publications (see [1] and references therein). They have the following orders of magnitude:

$$n \approx 1.1\text{–}1.6 \quad \text{and} \quad \chi \approx 10^{-10}\text{–}10^{-4},$$

that is, biological tissues are good dielectrics, for which the conductance of the medium can be, generally speaking, neglected in diffraction calculations. Correspondingly, they could be considered as a particular case of media with high conductance if conditions (15) and (16) can be met for them.

The applicability of the impedance boundary conditions to dielectric media is discussed in [12], and it is concluded that they are valid not only for media with high conductance but also for ideal dielectrics with a high real part of the refractive index. In this case, the angle of refraction of radiation at the medium–air interface can be considered as real and zero, which leads to conditions (15) and (16). In addition, as was shown in [12], for dielectrics, the errors in applying the impedance boundary conditions mostly affect the amplitude of the scattered field rather than the angular distribution of its intensity. The angular broadening of the radiation for such dielectrics, which is of primary interest to us, is adequately described by integral equations even under such oversimplified assumptions as Eq. (15). Moreover, Maradudin and Mendes [12] evidently did not notice that, at normal ($\varphi = 0$) illumination of the dielectric surface, the nonlocal angle of refraction of radiation at the interface is also real and nonzero, and the wave penetrating into the medium is homogeneous and transverse, so that conditions (15) and (16) remain effective in this case as well. Therefore, the solutions obtained above for conductors can also be used for estimating the scattering from dielectric media and biological tissues if the condition $\varphi = 0$ is satisfied (the case used most often in practice). Thus, according to Eqs. (13) and (18), biological tissues under normal illumination of their surface are characterized by a low surface reflection of the radiation (at a level of a few percent) and a wide reflection indicatrix (such as the Lambertian one), which is pointed out in most papers reporting the corresponding experimental data (clearly, only the purely boundary effects are meant).

To obtain the solution for the radiation propagating deep into the medium and derive the equation for its indicatrix, it is necessary to direct the radiation propagation vector $\boldsymbol{\omega}$ inside the medium (plate) in the existing computational scheme. It should also be taken into account that the medium of radiation propagation under the surface has a refractive index different from 1. As for the rest, the procedure for obtaining the solution is quite routine and similar to those described above. The angular scattering at high roughness for radiation penetrating into the medium is also described by the second term in Eq. (5), but with modified components of the vectors $\mathbf{q}$ and $\mathbf{k}$, and the vector product (17) transforms into the Fresnel transmission coefficient. This simple reasoning allows us to obtain an evaluative equation for the indicatrix of the radiation passing deep into a dielectric biological tissue based only on the available solutions (13) and (18) (without routine derivation),

$$\rho_\tau(\varphi = 0, \theta, \psi) = \frac{n^3(n-1)^2T^2}{h^2(n+1)^2(n\cos\theta - 1)^4} e^{-\frac{n^2\sin^2\theta T^2}{4h^2(n\cos\theta - 1)^2}}, \tag{19}$$

where ρ_τ is the transmission indicatrix and n is the real part of the refractive index of the medium.

Some doubts may arise about the fulfillment of the conditions of the far reception zone just above the surface and, correspondingly, the applicability of the concept of an indicatrix in this case. These doubts can be removed by a more detailed consideration of the procedures of integration of the equations when finding the statistical parameters of the scattered fields [16], as well as the issue concerning the position of the far zone in the presence of coarse roughness of the surface [25]. As was indicated in [16], in integration of the basic equations over the surface in order to find the incoherent component of the scattered field, the important domain of integration lies within the correlation length T. It is this domain that is the so-called dominant area [11] in formation of the diffracted field from the coarsely rough surface. The radiation coming from far elements of the surface is insignificant at the point of reception as compared to the radiation “distributed” within T. The distance to the far zone decreases drastically with the appearance of surface roughness. For coarsely rough surfaces, it does not exceed a few wavelengths [25], which is also comparable with T in the

Table 2. Transmission indicatrix calculated by Eq. (19) for dielectric surfaces at different observation angles and different parameters of the surface

θ, deg	Parameters of the surface						
	n = 1.4			*T*/*h* = 8.0			
	T/*h* = 4.0	*T*/*h* = 8.0	*T*/*h* = 16.0	*n* = 1.2	*n* = 1.6	*n* = 1.8	*n* = 2.0
0	47.64	190.6	762.2	571.2	107.7	74.39	56.89
5	34.29	43.57	1.773	6.411	46.44	41.17	35.80
10	11.41	0.325	0.000	0.001	3.060	6.234	8.263
15	1.150	0.001	0.000	0.000	0.016	0.176	0.541
20	0.012	0.000	0.000	0.000	0.001	0.001	0.006
25	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	Semispherical transmission coefficient						
	0.982	0.975	0.975	0.993	0.951	0.924	0.896

Kirchhoff approximation. Thus, as the radiation passes through the coarsely rough interface, the field is formed within the range of a few *T*, and all the above equations remain valid for analysis of purely surface effects (neglecting radiation scattering and absorption in the mass of the medium, which are subjects of RTT).

In Table 2, the degree of the beam broadening upon passage through a randomly rough surface of a dielectric medium is illustrated as a function of the angle θ and the parameters *n*, *T*, and *h*. This table summarizes the values of the indicatrix calculated by Eq. (19) for the set of absolute values of the parameters. The semispherical transmission coefficient is also calculated there for every case through integration of the indicatrix over the solid angle within the lower hemisphere. It is interesting that these data completely correlate with the data for reflection. Thus, if we take a dielectric surface ($\chi = 0$) with $T/h = 4$ and $n = 1.4$, then the integration of its reflection indicatrix (Table 1) over the upper hemisphere with allowance made for the Fresnel reflection coefficient by Eq. (18) gives the value of the normal hemispherical reflection coefficient $r = 0.027$, which in summing with the corresponding transmission coefficient $\tau = 0.982$ (Table 2) yields 1.009 (that is, the data for reflection and transmission correlate accurate to 1%, and no energy loss occurs at the interface).

The data of Table 2 clearly illustrate the beam broadening with increasing surface roughness and refractive index of the medium. True, in all the cases presented, which are closest to the real cases of biological tissues, the transmitted beam retains, to a high degree, its initial direction, and the angular divergence of the transmitted radiation proves not to be very high. Nevertheless, it can influence the general solution in RTT problems for the radiation field inside the medium as compared to the model of a parallel external flux and plane interface. In this case, the use of the boundary indicatrix in the form specified by Eq. (19) as a boundary condition for RTT problems can allow one to consider and study this effect.

CONCLUSIONS

Summarizing the paper, we can state that the solution of the problem of diffraction of electromagnetic waves on a randomly rough surface both with the boundary conditions of an ideal conductor and with the impedance boundary conditions allows an adequate analytical and closed description of the surface scattering of light by a rough interface. For the case of normal illumination of the surface and coarse roughness, evaluative analytical equations can also be obtained for dielectric materials and media, for example, biological tissues, in particular, for radiation propagating deep into the medium. In all cases, the solutions obtained can be formulated in terms of the photometric scattering indicatrix, which allows their direct application to computational RTT problems as boundary conditions. In addition, Eqs. (13) and (19) in combination can be used in RTT as an alternative to the well-known Henyey–Greenstein scattering phase function [3]. As to the basic question formulated at the beginning of this paper about the scattering of radiation propagating into a medium, the calculations have shown that, in the general case of dielectric surfaces, a light beam having passed through a rough interface retains, to a high degree, its initial direction, while the angular divergence of the transmitted beam is not very high.

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APPENDIX 1

EXPLANATIONS TO EQ. (10) FOR DETERMINATION OF THE SOLID ANGLE

The form of Eq. (10) differs somewhat from the classical photometric equations for determination of the solid angle; therefore, it requires additional explanations. Very often, the mathematical operations with angles are performed so that the angles used in the equations seem to be dimensionless. For example, in photometry the purely mathematical definition of the solid angle is often formulated (example from [19]) as follows: $d\Omega = dA/R^2$, whence the parameter $d\Omega$ seems to be dimensionless, though it is measured in steradians, because the division of square meters by square meters gives a dimensionless value for the angle. However, even if an ordinary plane angle is considered, everybody knows that there are different systems of measures for plane angles: in units (fractions) of arc, in radians (radian measure of angle), in degrees (degree measure), and so on. These systems of measures are, generally speaking, physical; that is, they have conversion coefficients from one system to another, terminology, and derivative parameters (for example, minutes or seconds of arc). Therefore, physically, the angle, completely in accordance with the classical definition of any measurable parameter [26], should be classified as a dimensional parameter. For example, it should be subject to measurement. Using the corresponding instrument (protractor), we can obtain the value of a drawn angle, in this case, in degrees. However, what is the confusion here? Should the solid angle be considered as a dimensional or dimensionless parameter in physical calculations?

The general equation for any dimensional physical parameter is formulated according to the classical theory of dimension in the following way [26]:

$$G = N_G[G], \qquad \dim G \equiv \dim[G], \tag{A1}$$

where G is the parameter to be measured, $N_G \equiv \{G\}$ is the amount in which some reference (unit) is contained in G, and [G] is some reference or unit.

At the same time, from elementary plane geometry, we know [27] that the value of the central angle in a circle is proportional to the ratio of the arc length within this angle to the radius. Thus, taking into account Eq. (A.1), we have the following general form of the equation for determining an arbitrary central angle α:

$$\alpha = k_0 \frac{N_S[S]}{N_R[R]}, \tag{A2}$$

where N_S and N_R are the numerical values of the arc length and the radius, respectively; $[S]$ and $[R]$ are the units in which they are measured; and k_0 is the coefficient of proportionality.

In this case, it becomes clear that it is the coefficient k_0 that determines the desired system of measure for angles in any particular problem. At $k_0 = 1/2\pi$, the angle α is measured in fractions of arc; at $k_0 = 1$, the angle is measured in radians; and at $k_0 = 180/\pi$, it is measured in degrees. Comparing Eqs. (A.2) and (A.1), canceling the units of the arc length and radius ($[S] = [R]$), and taking into account that $N_\alpha = N_S/N_R$, we can definitely assert that the coefficient of proportionality k_0 in (A.2) plays the role of a reference unit for measurement of the angle, that is, $k_0 \equiv [\alpha]$. True, it cannot be used directly to measure the angle. For this purpose, by definition, the angle with the arc length equal to the radius is used instead of this coefficient [27]. But this is precisely the physical meaning of the coefficient k_0. It determines the units of this reference angle, and, in principle, it cannot be excluded from the physical equations. It is quite another matter that $k_0 \equiv [\alpha] = 1$ rad in the radian system of measure, and this coefficient is almost always omitted for brevity. However, it should always be present in the general physical equations.

Similarly, in electrodynamics, the issue concerning the measurement and dimensions of the material parameters of a medium (permittivity ε and permeability μ) and "equalization" of different units in the Maxwell equations was discussed in due course. The way out was also found by invoking the theory of dimension through introduction of special dimensional coefficients ε_0 and μ_0 for a vacuum. These coefficients also cannot be used to measure the sought parameters, but they form the basic dimensional system of measures, needed for any problem of electrodynamics, and appear in all basic equations (for more details, see [28]).

Thus, in physical calculations, any angle should be considered with regard to the units in which it is measured. In this case, the final equation for determination of the angle α in the example with a circle with the designation $k_0 \equiv [\alpha] \equiv \alpha_0$ should be formulated as follows:

$$\alpha = \frac{S}{R}\alpha_0,$$

where $S = N_S[S]$ is the arc length and $R = N_R[R]$ is the radius. With this, the final equation for the solid angle in the form (10) should no longer raise questions, in spite of the fact that the only established system of measures for solid angles is the radian (rather, steradian) system with $[\Omega] = \Omega_0 = 1$ sr.

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